On Fractional Transportation problem

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Abstract: In this paper investigates the multi objective linear fractional transportation MOLFT problem where the cost coefficients are represented by fuzzy rough parameters. We present an algorithm for solving a fuzzy rough multi objective linear fractional transportation (FRMOLFT) problem. The FRMOLFT problem reduced to Multi objective linear fractional transportation (MOLFT) problem based on the ranking function for all fuzzy rough parameters. The Dinkelbach,s and weighting method is used for solving MOLFT problem. The algorithm is explained in detail with an example.

Keywords: *Linear fractional Transportation problem, Robust ranking function, Triangular fuzzy rough number.*

Introduction

In the real life, there are many problems where we need to optimize profit/cost, profit/manpower requirement, adept/equity, nurse/patient etc., and the linear fractional transportation comes into a picture. The linear fractional programming problem seeks to optimize the objective function of non-negative variables of quotient from with linear functions in the numerator and denominator subject to a set of linear and homogeneous constraints. The fractional transportation problem (FTP) plays an important role in logistics and supply management for reducing cost and improving service. In the real world, however, the parameters in the models are seldom known exactly and have to be estimated. Schaible and Shi [13], and many other researchers worked on linear fractional programming problem, Bajalinov [3] formulated linear fractional transportation problem in general form and shortly overviewed its main theoretical results, Ammar and Muamer [2] introduced algorithm for solving multi objective linear fractional programming problems with a fuzzy rough coefficient in the objective functions.

A classical transportation problem is a minimization problem of the cost of transportation from some origins to some other destinations. The minimum cost planning plays an important role in solving the transportation problem from origins to different destinations, such as from factories to warehouses or from warehouses to supermarkets, etc. Here we are considering a class of transportation problem called linear fractional transportation (LFT) problem, which is similar to the classical transportation problem except the objective function is a ratio of two linear functions. These types of the problems arise when we want to minimize the cost to time ratio or maximize the profit to time ratio. Joshi and Gupta [8] obtained the initial basic feasible solution for the linear fractional transportation problem. Sirvi et al. [14] proposed a solution method for linear fractional programming transportation problem. Khurana and Arora [10], Jain and Arya [9], Monta [11], and many others have worked on different types of fractional transportation problem.

Fuzzy sets introduced by Zadeh in (1965). A fuzzy transportation problem is an extension of a linear transportation problem, where at least one of the transportation costs, supply and demand quantities are fuzzy quantities. The objective function of the fuzzy transportation problem is to determine the total fuzzy minimum transportation cost by shipping the fuzzy supply and fuzzy demand. Bellman and Zadeh [4], developed further. Dutta and Murthy [6] investigated the transportation problem with additional impurity restrictions where costs are not deterministic numbers but imprecise ones, also the elements of the cost matrix are subnormal fuzzy intervals with strictly increasing linear membership functions. Cetin and Tiryaki [5] a fuzzy approach using generalized Dinkelbach, s algorithm for solving multi objective linear fractional transportation problem. Yao and Wu [15] proposed a signed distance ranking to rank Triangular fuzzy number. In this paper, proposed an algorithm for solving a fuzzy rough multi objective linear fractional transportation (FRMOLFT) problem. The proposed algorithm is explained in detail with an example.

Preliminaries

In this section, some basic definitions are presented.

Triangular Fuzzy Number: [1, 2, 4]

Definition 1. For any a fuzzy set $\tilde{A}$ the membership function of $\tilde{A}$ is written as $\mu_{\tilde{A}}(x)$, a fuzzy set $\tilde{A}$ is defined by:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \epsilon \mathcal{R}^n \ , \mu_{\tilde{A}}(x) \epsilon [0,1] \ \}$$

The $(\alpha - cut)$ of fuzzy set $\tilde{A}$ defined as:

$A_\alpha = \{x\colon \mu_{\tilde{A}}(x) \geq \alpha ,\} = [a_\alpha^L \ , \ a_\alpha^U],\ \alpha \epsilon [0,1]$ Where

$a_\alpha^L = inf\{x\colon A(x) \geq \alpha \}\ and \ \ a_\alpha^U = sup\{\ x\colon A(x) \geq \alpha \},\ \alpha \epsilon [0,1]$.

Definition 2. A fuzzy set $\widetilde{A}$ is convex if for any

$x_1 , x_2 \in \mathcal{R}^n \ and \ \omega \in [0,1]$, we have:

$$\mu_{\tilde{A}}(\omega x_1 + (1-\omega)x_2) \geq min\{\mu_{\tilde{A}}(x_1), \ \mu_{\tilde{A}}(x_2)\}.$$

Definition 3. Let there exists$a_1, a_2, a_3 \in \mathcal{R}$ such that:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ , & \\ \frac{x-a_3}{a_2-a_3} & a_2 \leq x \leq a_3 \\ , & \\ 0 & otherwise \end{cases}$$

Then we say that $\tilde{A}$ is triangular fuzzy number, written as: $\tilde{A} = (a_1, a_2, a_3)$

In this paper the class of all triangular fuzzy number is called Triangular fuzzy number space, which is denoted by $TF(N)$.

Definition 4. For any triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$, for all $\alpha\epsilon[0,1]$ we get a crisp interval by $\alpha - cut$ operation defined as:

$\tilde{A}_\alpha = [a_1 + (a_2 - a_1)\alpha \ , \ a_3 + (a_2 - a_3)\alpha] = [a_1^L(\alpha), a_3^U(\alpha)]$.

Definition 5. For any two fuzzy numbers $\tilde{A}$, $\tilde{B} \in TF(N)$ we say that $\tilde{A} \subseteq \tilde{B}$ iff $\mu_{\tilde{A}}(x) \precsim \mu_{\tilde{B}}(x)$ *for all* $x\epsilon\mathcal{R}$.

Basic Operations of Triangular Fuzzy Number: [2, 4, 14]

Let $\tilde{A} = (a_1, a_2, a_3)$ *and* $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, where $a_1, a_2, a_3, b_1, b_2, b_3 \in \mathcal{R}$. Then the arithmetic operations are defined by

$\tilde{A} + \tilde{B} = (a_1 + b_1 , \ a_2 + b_2 , \ a_3 + b_3)$

$\tilde{A} - \tilde{B} = (a_1 - b_3 , \ a_2 - b_2 , \ a_3 - b_1)$,

$k\tilde{A} = (k\, a_1 , \ ka_2 , \ ka_3), \quad for\ k \geq 0$.

$k\tilde{A} = (k\, a_3 , \ ka_2 , \ ka_1), \quad for\ k < 0$.

$$\tilde{A}\,.\,\tilde{B} = \begin{cases} (a_1 b_1 , & a_2 b_2 , & a_3 b_3) & a_1 \geq 0 \\ (a_1 b_3 , & a_2 b_2 , & a_3 b_3) & a_1 < 0 , a_3 \geq 0 \\ (a_1 b_3 , & a_2 b_2 , & a_3 b_1) & a_3 < 0 \end{cases}$$

$If\ 0 \notin \tilde{B} = (b_1, b_2, b_3) \quad then \quad \tilde{A} \div \tilde{B} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$.

Triangular Fuzzy Rough Number: [2]

A triangular fuzzy rough number denoted by $\tilde{A}^R$ define as:

$\tilde{A}^R = [\tilde{A}^L : \tilde{A}^U] = [(a_1^L, a_2, a_3^L) : (a_1^U, a_2, a_3^U)]$ is fuzzy rough set *with* a_1^U, a_1^L, a_2, a_3^L *and* $a_3^U \in \mathcal{R}$ *such that* $a_1^U \leq a_1^L \leq a_2 \leq a_3^L \leq a_3^U$ where the membership functions can be defined as:

$$\mu_{\tilde{A}^L}(x) = \begin{cases} \frac{x-a_1^L}{a_2-a_1^L} & a_1^L \le x \le a_2, \\ \frac{x-a_3^L}{a_2-a_3^L} & a_2 \le x \le a_3^L, \\ 0 & otherwise \end{cases} \quad , \quad \mu_{\tilde{A}^U}(x) = \begin{cases} \frac{x-a_1^U}{a_2-a_1^U} & a_1^U \le x \le a_2, \\ \frac{x-a_3^U}{a_2-a_3^U} & a_2 \le x \le a_3^U, \\ 0 & otherwise \end{cases}$$

Note that $\tilde{A}^L = (a_1^L, a_2, a_3^L)$, $\tilde{A}^U = (a_1^U, a_2, a_3^U)$ *and* $\tilde{A}^L \subseteqq \tilde{A}^U$.

The class of all triangular fuzzy rough numbers is called Triangular fuzzy rough number space.

Definition 6. For any triangular fuzzy rough number $\tilde{A}^R = [(a_1^L, a_2, a_3^L) : (a_1^U, a_2, a_3^U)]$, *for all* $\alpha \epsilon [0,1]$ we get a rough interval by $\alpha - cut$ operation defined as:

$$(\tilde{A}^R)_\alpha = [(\tilde{A}^L)_\alpha : (\tilde{A}^U)_\alpha] \text{ where}$$

$$(\tilde{A}^L)_\alpha = [a_1^L + (a_2 - a_1^L)\alpha \ , a_3^L + (a_2 - a_3^L)\alpha] = [a_1^L(\alpha), a_3^L(\alpha)].$$

$$(\tilde{A}^U)_\alpha = [a_1^U + (a_2 - a_1^U)\alpha \ , a_3^U + (a_2 - a_3^U)\alpha] = [a_1^U(\alpha), a_3^U(\alpha)].$$

Basic Operations for Triangular Fuzzy Rough Number: [2]

Let $\tilde{A}^R = [(a_1^L, a_2, a_3^L) : (a_1^U, a_2, a_3^U)]$ and $\tilde{B}^R = [(b_1^L, b_2, b_3^L) : (b_1^U, b_2, b_3^U)]$ be two triangular fuzzy rough numbers, where $\tilde{A}^R$ *and* $\tilde{B}^R \ge \tilde{0}^R$, then the arithmetic operations are defined by

1) $\tilde{A}^R + \tilde{B}^R = [(\tilde{A}^L + \tilde{B}^L) : (\tilde{A}^U + \tilde{B}^U)]$

Where $\tilde{A}^L + \tilde{B}^L = (a_1^L + b_1^L,\ a_2 + b_2,\ a_3^L + b_3^L)$

$\tilde{A}^U + \tilde{B}^U = (a_1^U + b_1^U,\ a_2 + b_2,\ a_3^U + b_3^U)$

2) $\tilde{A}^R - \tilde{B}^R = [(\tilde{A}^L - \tilde{B}^L) : (\tilde{A}^U - \tilde{B}^U)]$

Where $\tilde{A}^L - \tilde{B}^L = (a_1^L - b_3^L,\ a_2 - b_2,\ a_3^L - b_1^L)$

$\tilde{A}^U - \tilde{B}^U = (a_1^U - b_3^U, a_2 - b_2, a_3^U - b_1^U)$

3) $\tilde{A}^R \times \tilde{B}^R = [(\tilde{A}^L \times \tilde{B}^L) : (\tilde{A}^U \times \tilde{B}^U)]$

Where $\tilde{A}^L \times \tilde{B}^L = (a_1^L \times b_1^L, a_2 \times b_2, a_3^L \times b_3^L)$

$\tilde{A}^U \times \tilde{B}^U = (a_1^U \times b_1^U, a_2 \times b_2, a_3^U \times b_3^U)$

4) *If* $0 \notin \tilde{B}^R$ *then* $\tilde{A}^R \div \tilde{B}^R = [(\tilde{A}^L \div \tilde{B}^L) : (\tilde{A}^U \div \tilde{B}^U)]$

Where $\tilde{A}^L \div \tilde{B}^L = (a_1^L \div b_3^L,\ a_2 \div b_2,\ a_3^L \div b_1^L)$

$\tilde{A}^U \div \tilde{B}^U = (a_1^U \div b_3^U,\ a_2 \div b_2,\ a_3^U \div b_1^U)$

A Ranking Function for Triangular Fuzzy Rough Number : [2, 15]

Assume that $\Re : \tilde{F}^R \longrightarrow \mathcal{R}$ be linear ordered function that maps each triangular fuzzy rough numbers in to the real number in $\mathcal{R}$, which $\tilde{F}^R$ denotes the Triangular fuzzy rough number space. The ranking function for a fuzzy rough, $\tilde{A}^R = [\tilde{A}^L : \tilde{A}^U]$ where, $\tilde{A}^L = (a_1^L, a_2, a_3^L)$, $\tilde{A}^U = (a_1^U, a_2, a_3^U)$ can be defined using convex combination between $\Re(\tilde{A}^L)$, $\Re(\tilde{A}^U)$ as suggested by Reuben's to get:

$$\Re(\tilde{A}^R) = \frac{1}{8}(a_1^L + a_1^U + 4a_2 + a_3^L + a_3^U)$$

Accordingly for any two fuzzy rough $\tilde{A}^R$ and $\tilde{B}^R$ we have:

1. $\tilde{A}^R \gtrsim^R \tilde{B}^R$ *iff* $\Re(\tilde{A}^R) \geq \Re(\tilde{B}^R)$
2. $\tilde{A}^R \lesssim^R \tilde{B}^R$ *iff* $\Re(\tilde{A}^R) \leq \Re(\tilde{B}^R)$
3. $\tilde{A}^R \cong \tilde{B}^R$ iff $\Re(\tilde{A}^R) = \Re(\tilde{B}^R)$.

Problem formulation

Let $(\tilde{c}_{ij}^R)^k$, $(\tilde{d}_{ij}^R)^k$, $(\tilde{c}_0^R)^k$ and $(\tilde{d}_0^R)^k$ denote the triangular fuzzy rough numbers, then the fuzzy rough multi objective linear fractional transportation (FRMOLFT) problem formulation following as:

$$\left.\begin{aligned}
&Min\ \ \tilde{Z}_k^R(x) = \frac{\tilde{N}_k^R(x)}{\tilde{D}_k^R(x)} = \frac{\sum_{i=1}^m \sum_{j=1}^n (\tilde{c}_{ij}^R)^k\ x_{ij} + (\tilde{c}_0^R)^k}{\sum_{i=1}^m \sum_{j=1}^n (\tilde{d}_{ij}^R)^k\ x_{ij} + (\tilde{d}_0^R)^k},\\
&subject\ to: \quad \sum_{j=1}^n x_{ij} = A_i \quad , \quad i = 1,2,\dots,m,\\
&\qquad\qquad\qquad \sum_{i=1}^m x_{ij} = B_j \quad , \quad j = 1,2,\dots,n,\\
&\qquad\qquad\qquad x_{ij} \geq 0 \quad for\ all\ i,j
\end{aligned}\right\} \quad (1)$$

We suppose that $\tilde{D}_k^R(x) > 0$, $k = 1,2,\dots,K$ *and* $\forall x = (x_{ij}) \in S$,

where $S \neq \phi$ denotes a convex and compact feasible set defined by constraints; the functions $\tilde{N}_k^R(x)$ *and* $\tilde{D}_k^R(x)$ are continuous on S. Further, we assume that

$$A_i\ , B_j > 0\ \ \forall i, j\ ;\ (\tilde{c}_{ij}^R)^k,\ (\tilde{d}_{ij}^R)^k,\ (\tilde{c}_0^R)^k, (\tilde{d}_0^R)^k > 0\ \forall\ i,j$$

and suppose that $\sum_{i=1}^{m} A_i = \sum_{j=1}^{n} B_j$ (2)

The equality (2) is treated as a necessary and sufficient condition for the existence of a feasible solution to problems Model (1).

Definition 7. (Efficient Solution of FRMOLFT Problem)

A point $x^* \in \mathcal{R}^n$ is an efficient solution of FRMOLFT problem Model (1) if there does not exist $x \in \mathcal{R}^n$ such that

$$\frac{\tilde{N}_k^R(x^*)}{\tilde{D}_k^R(x^*)} \gtrsim^R \frac{\tilde{N}_k^R(x)}{\tilde{D}_k^R(x)},\quad k = 1,2,\dots,K \ and \ \frac{\tilde{N}_k^R(x^*)}{\tilde{D}_k^R(x^*)} \gtrsim^R \frac{\tilde{N}_k^R(x)}{\tilde{D}_k^R(x)}$$

for at least one k.

Algorithm Solution For (FRMOLFT) Problem

Step 1: Using a Ranking function for all triangular fuzzy rough coefficients we can reduce the FRMOLFT problem into the multi objective linear fractional transportation (MOLFT) problem as follows:

$$\left.\begin{aligned} Min\ \Re(\tilde{Z}_k^R(x)) &= \frac{\Re(\tilde{N}_k^R(x))}{\Re(\tilde{D}_k^R(x))} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} \Re(\tilde{c}_{ij}^R)^k\, x_{ij} + \Re(\tilde{c}_0^R)^k}{\sum_{i=1}^{m}\sum_{j=1}^{n} \Re(\tilde{d}_{ij}^R)^k\, x_{ij} + \Re(\tilde{d}_0^R)^k}, \\ subject\ to: &\quad \sum_{j=1}^{n} x_{ij} = A_i, \quad i = 1,2,\dots,m, \\ &\quad \sum_{i=1}^{m} x_{ij} = B_j, \quad j = 1,2,\dots,n, \\ &\quad x_{ij} \ge 0 \quad for\ all\ i,j \end{aligned}\right\} \quad (3)$$

Now to simplify the MOLFT problem in Model (3) can be written as follows:

$$\left.\begin{aligned} &Min\ \Re(\tilde{Z}_k^R(x)) = \frac{\Re\,(\tilde{N}_k^R(x))}{\Re\,(\tilde{D}_k^R(x))} = \frac{\sum_{i=1}^{m} f_i^k + \Re(\tilde{c}_0^R)^k}{\sum_{i=1}^{m} g_i^k + \Re(\tilde{d}_0^R)^k} \quad , k = 1,2,\ldots K \\ &subject\ to: \quad \sum_{j=1}^{n} x_{ij} = A_i \quad , \quad i = 1,2,\ldots,m \\ &\qquad \sum_{i=1}^{m} x_{ij} = B_j \quad , \quad j = 1,2,\ldots,n \\ &\qquad x_{ij} \geq 0 \quad for\ all\ i,j \end{aligned}\right\} \quad (4)$$

Step 2: The MOLFT problem in Model (4) can be reduced to the linear transportation (LT) problem with Dinkelbach **[see 5]** as follows form:

$$\left.\begin{aligned} &Min\ \ Z(x) = \sum_{k=1}^{K} w_k \left\{\Re\left(\tilde{N}_k^R(x)\right) - \left(\tilde{Z}_k^R(x)\right)^* (\Re\,(\tilde{D}_k^R(x))\right\} \\ &subject\ to: \quad \sum_{j=1}^{n} x_{ij} = A_i \quad , \quad i = 1,2,\ldots,m \\ &\qquad \sum_{i=1}^{m} x_{ij} = B_j \quad , \quad j = 1,2,\ldots,n \\ &\qquad x_{ij} \geq 0 \quad for\ all\ i,j \quad k = 1,2,\ldots,K \end{aligned}\right\} \quad (5)$$

Where $(\tilde{Z}_k^R(x))^*$ minimum values of each objective function for the MOLFT problem in Model (4).

Step 3: solving the linear programming problem in Model (5) for any $w_k \in [0,1]$, $\sum_{k=1}^{K} w_k = 1$, the optimal solution thus obtained shall be efficient solution of the problem in Model (1).

Theorem

If $\bar{x}$ is an optimal solution of the problem in Model (5) Then $\bar{x}$ is an efficient solution of the problem in Model (1).

The proof of this theorem is much similar to the proof given by Guzel in [7].

Example.

To illustrate the Algorithm Solution, consider there are two supplies and two demands nods in this example, *where the transportation costs, supply and demand quantities are* triangular fuzzy rough numbers, then the following FRMOLFT problem can be writhing as:

$$Min\ \tilde{Z}_1^R(x) = \frac{\tilde{2}^R x_{11} + \tilde{3}^R x_{12} + \tilde{5}^R x_{21} + \tilde{8}^R x_{22}}{\tilde{1}^R x_{11} + \tilde{2}^R x_{12} + \tilde{3}^R x_{21} + \tilde{4}^R x_{22}}$$

$$Min\ \tilde{Z}_2^R(x) = \frac{\tilde{3}^R x_{11} + \tilde{2}^R x_{12} + \tilde{1}^R x_{21} + \tilde{7}^R x_{22}}{\tilde{2}^R x_{11} + \tilde{3}^R x_{12} + \tilde{6}^R x_{21} + \tilde{5}^R x_{22}}$$

Subject to:
$$\begin{cases} x_{11} + x_{12} = 200 \quad , \quad x_{21} + x_{22} = 350 \\ , \\ x_{11} + x_{21} = 250 \quad , \quad x_{12} + x_{22} = 300 \\ , \\ x_{11},\ x_{12},\ x_{21},\ x_{22} \geq 0 \end{cases}$$

$$where \begin{cases} \tilde{1}^R = [(0,1,\ 2) : (0,1,4)], \quad \tilde{2}^R = [(1,2,\ 4) : (0,2,\ 5)], \\ \tilde{3}^R = [(1,3,\ 5) : (1,\ 3,\ 6)], \quad \tilde{4}^R = [(3,4,6) : (2,4,8)] \\ \tilde{5}^R = [(2,5,7) : (1,5,\ 9)], \quad \tilde{6}^R = [(3,\ 6,8) : (2,6,10)] \\ \tilde{7}^R = [(6,7,8) : (5,7,10)], \quad \tilde{8}^R = [(6,8,10) : (4,8,12)]. \end{cases}$$

Solution:

To solve the problem, we have to determine the ranking index of the all coefficients. For example the $\alpha - cut$ of fuzzy rough cost

$\tilde{4}^R = [(3,4,6) : (2,4,8)]$, *is*

$(\tilde{4}^R)_\alpha = \left[[3+\ \alpha, 6-2\alpha] : [2+2\alpha,\ 8-4\alpha]\right]$ *for which*

$$\Re(\tilde{4}^R) = \frac{1}{8}\,(3+6+4(4)+2+8) = 4.38$$

Similarly, the ranking for the fuzzy rough costs are calculated as follows:

$$\begin{cases} \Re(\tilde{1}^R) = 1.25,\ \Re(\tilde{2}^R) = 2.25,\ \Re(\tilde{3}^R) = 3.25,\ \Re(\tilde{4}^R) = 4.38 \\ , \\ \Re(\tilde{5}^R) = 4.88,\ \Re(\tilde{6}^R) = 5.88,\ \Re(\tilde{7}^R) = 7.25,\ \Re(\tilde{8}^R) = 8 \end{cases}$$

Using the value of ranking for all coefficients in the FRMOLFT problem we have:

$$\begin{cases} Min\ \mathcal{R}\left(\tilde{Z}_1^R(x)\right) = \dfrac{2.25\,x_{11} + 3.25\ x_{12} + 4.88\ x_{21} + 8\,x_{22}}{1.25\,x_{11} + 2.25\,x_{12} + 3.25\ x_{21} + 4.38\ x_{22}} \\ , \\ Min\ \Re(\tilde{Z}_2^R(x)) = \dfrac{3.25\ x_{11} + 2.25\ x_{12} + 1.25\ x_{21} + 7.25\,x_{22}}{2.25\,x_{11} + 3.25\ x_{12} + 5.88\ x_{21} + 4.88\ x_{22}} \end{cases}$$

$$\text{Subject to: } \begin{cases} x_{11} + x_{12} = 200 \quad , \quad x_{21} + x_{22} = 350 \\ , \\ x_{11} + x_{21} = 250 \quad , \quad x_{12} + x_{22} = 300 \\ , \\ x_{11}, x_{12}, x_{21}, x_{22} \geq 0 \end{cases}$$

Now solve for each objective function we have:

$\Re\,(\tilde{Z}_1^R(x))^* = 1.57 \;\; , \;\; \Re(\tilde{Z}_2^R(x))^* = 0.57$

Using Dinkelbach and weighting method to reduce the MOLFT problem for the linear transportation LT problem as follows form:

$$\begin{cases} Min\ Z = w_1(0.2875\,x_{11} - 0.2825\,x_{12} - 0.2225\,x_{21} + 1.1234\,x_{22}) \\ , \\ + w_2(1.9675\,x_{11} + 0.3975\,x_{12} - 2.1016\,x_{21} + 4.4684\,x_{22}) \end{cases}$$

$$\text{Subject to: } \begin{cases} x_{11} + x_{12} = 200 \quad , \quad x_{21} + x_{22} = 350 \\ , \\ x_{11} + x_{21} = 250 \quad , \quad x_{12} + x_{22} = 300 \\ , \\ x_{11}, x_{12}, x_{21}, x_{22} \geq 0 \end{cases}$$

For choose values of weighting $w_1 = w_2 = 0.5$ we have the following problem

$$Min\ Z = 1.1275\,x_{11} + 0.0575\,x_{12} - 1.1621\,x_{21} + 2.7959\,x_{22}$$

$$\text{Subject to: } \begin{cases} x_{11} + x_{12} = 200 \quad , \quad x_{21} + x_{22} = 350 \\ , \\ x_{11} + x_{21} = 250 \quad , \quad x_{12} + x_{22} = 300 \\ , \\ x_{11}, x_{12}, x_{21}, x_{22} \geq 0 \end{cases}$$

The optimal solution of the above linear transportation problem is:

$$x_{11} = 0, \quad x_{12} = 200, \quad x_{21} = 250, \quad x_{22} = 100$$

Then the efficent solution of FRMOLFT problem is:

$$x_{11} = 0, \quad x_{12} = 200, \quad x_{21} = 250, \quad x_{22} = 100$$

With the minimum objective value are:

$$\begin{cases} \tilde{Z}_1^R(x) = [\,(0.57,\ 1.7,\ 3.75) : (0.26,\ 1.7,\ 10.33)\,] \\ \tilde{Z}_2^R(x) = [\,(0.22,\ 0.52,\ 1.56) : (0.11,\ 0.52,\ 3.53)\,] \end{cases}$$

Conclusion

A fuzzy rough multi objective linear fractional transportation (FRMOLFT) problem is discussed in this paper. The FRMOLFT problem reduced to Multi objective linear fractional transportation (MOLFT) problem based on rank- ing function for all fuzzy rough parameters. The Dinkelbach algorithm and weighting method is used for solve MOLFT problem. The algorithm solution has been illustrated by example.

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